Left Ventricular Length-Force-Shortening Relations Before and After Surgical Correction of Chronic Mitral Regurgitation

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Objectives. We tested the hypothesis that postoperative left ventricular (LV) systolic wall stress can be predicted from the change in LV diastolic dimension and ejection fraction (EF) after surgical correction of chronic mitral regurgitation (MR). We used a simple mathematic model to predict postoperative systolic stress from end-diastolic dimension and EF. The validity of this model was assessed using data from 21 patients undergoing mitral valve replacement (MVR) for chronic MR.

Background. The decline in EF after MVR for chronic MR is traditionally thought to be a consequence of a postoperative increase in afterload, caused by closure of a low resistance runoff into the left atrium. However, consideration of the Laplace relation suggests that afterload does not necessarily increase after the operation.

Methods. A spherical mathematical model of the left ventricle was used to define the relations between LV end-diastolic dimension, systolic wall stress and EF. To test the validity of this model, clinical and echocardiographic data were obtained from 21 patients with chronic MR before and 10 to 14 days after MVR. These echocardiographic data were examined with reference to plots derived from the mathematic model.

Results. Patients were categorized as those in whom end-diastolic dimension declined after the operation (group I, n = 15) and those with no reduction in end-diastolic dimension (group II, n = 6). Group I patients were subclassified into those undergoing MVR with chordal preservation (group Ia) and those undergoing MVR with chordal transection (group Ib). In groups Ib and II, there were significant reductions in EF (56 ± 3% to 48 ± 3% in group Ib and 50 ± 2% to 40 ± 3% in group II, both p < 0.05), but the changes in end-diastolic dimension and wall stress differed. In group Ib, end-diastolic dimension decreased and systolic wall stress was unchanged; in group II, end-diastolic dimension was unchanged and wall stress increased. In contrast, group Ia patients experienced a substantial reduction in end-diastolic dimension, no change in EF and a reduction in stress. The corresponding length-force-shortening coordinates closely approximate those predicted from a mathematic model relating end-diastolic dimension to EF and systolic wall stress.

Conclusions. Concordant echocardiographic and mathematical model results indicate that postoperative changes in systolic stress are directly related to changes in chamber size and that LV afterload may fall when chordal preservation techniques are used in combination with MVR.

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simple mathematical model relating LV end-diastolic dimension, EF and systolic wall stress. This length-force-shortening model was used to predict postoperative end-systolic stress from postoperative end-diastolic dimension and EF. The validity of this model was tested using data from patients undergoing mitral valve replacement (MVR) for chronic MR.

**Mathematical Model**

A spherical model of the LV was used to define the relations between LV end-diastolic dimension, end-systolic wall stress and EF (Fig. 1). In this model, end-systolic stress was calculated over a range of specified end-diastolic dimensions and EFs.

**Assumptions.** For the purpose of the model, we assumed that LV mass remained constant throughout the cardiac cycle, that mass did not change within the first 2 weeks after the operation and that LV mass was 200 g. Left ventricular mean systolic pressure was assigned a value of 100 mm Hg. The LV was modeled as a sphere. These values of mass and pressure closely approximated the values found in the patient group reported subsequently.

**Calculations.** The determinants of end-systolic wall stress include end-systolic dimension, end-systolic wall thickness and end-systolic pressure. Each of these three values was calculated over a range of LV end-diastolic dimensions at three specific EFs using the methods described subsequently.

**End-systolic dimension (LVIDs).** Left ventricular end-diastolic dimension (LVIDd) and EF were specified and the end-diastolic volume and end-systolic volume were computed using the Teichholz method (7):

\[
\text{Volume (ml) = } (7(2.4 + \text{LVID})) \times \text{LVID}^3.
\]  

[1]

Thus, end-diastolic volume was calculated from a specified LVIDd; end-systolic volume was calculated from a specified end-diastolic volume; and EF was calculated using the equation

\[
\text{EF} (%) = (\text{EDV} - \text{ESV})/\text{EDV},
\]  

[2]

where EDV = end-diastolic volume (ml) and ESV = end-systolic volume (ml); and LVIDd was calculated from end-diastolic volume and the Teichholz formula (Equation 1).

**End-systolic wall thickness (Ths).** Both end-diastolic wall thickness (Thd) and Ths were calculated using the formula for LV mass (8):

\[
\text{LV mass}_s = (\text{LVID}_s + 2\text{Ths})^3 - (\text{LVID}_{s/4})^3 = \text{LV mass},
\]  

\[
= (\text{LVID}_s + 2\text{Ths})^3 - (\text{LVID}_{s/4})^3 = 200 \text{ g}. \]  

[3]

Thus, end-systolic wall thickness was calculated over a range of end-diastolic dimensions. As stated earlier, we assumed that LV mass was 200 g and did not change throughout the cardiac cycle, and the LV was remodeled as a sphere.

**End-systolic wall stress (σs).** End-systolic wall stress was calculated from end-diastolic dimension, end-systolic thickness and mean arterial pressure (MAP). End-systolic stress values were calculated for end-diastolic dimensions ranging from 40 to 70 mm and EFs ranging from 35% to 75% using the formula

\[
\sigma_s (\text{g/cm}^2) = \text{MAP} \times \text{LVID}_s/4\text{Ths} (1 + \text{Ths}/\text{LVID}_s).
\]  

[4]

This formula is equivalent to that used in previous studies (2,9–11).

**Hypothesis.** The curves plotted in Figure 1, derived from the mathematic model, demonstrate the interdependence of end-systolic stress, EF and end-diastolic dimension. These curves provide the format in which to present the hypothesis proposed and tested in this study. After MVR, there are three possible mechanical outcomes illustrated schematically by the three arrows labeled A, B and C. If end-diastolic dimension remains constant and the EF falls, the model predicts that end-systolic wall stress will increase (arrow A). In contrast, if end-diastolic dimension decreases and EF remains constant, the model predicts that end-systolic stress will decrease (arrow C).
Patient Data

To test the validity of the mathematic model and to test the proposed hypothesis, clinical and echocardiographic data were obtained from 21 patients with chronic MR before and after MVR. These measured clinical data were plotted with reference to data derived from the mathematical model.

Mitral valve replacements were performed both at the University of Massachusetts and the Medical University of South Carolina. The decision to perform mitral valve surgery was made by physicians not involved in this study and was based on clinical, echocardiographic, hemodynamic and angiographic criteria. The decision regarding the type of corrective surgery performed was made by the cardiovascular surgeon on the sole basis of the intraoperative anatomic status of the mitral valve. In each patient the mitral apparatus was preserved when possible, but ablated when necessary. In the current study we chose to examine only two groups of patients: those undergoing MVR with complete chordal transection and those undergoing MVR with chordal preservation. Eight patients (four from each institution) underwent MVR with a single type of chordal preservation technique—that of preservation of the posterior leaflet and its chordal structures. Although this study ignores a variety of other surgical techniques used to correct chronic MR while preserving the mitral apparatus, the experimental design of the present study has the advantage that both groups of patients received a mitral valve prosthesis; the only difference between the groups was the preservation or transection of the chordal attachments. The criteria for inclusion into this study were based on clinical characteristics, including the absence of coronary artery disease and regional wall motion abnormalities, the presence of normal sinus rhythm and the presence of isolated, severe, chronic MR, as well as echocardiographic characteristics, including very high quality study, multiple views of the LV and adequate preoperative and early postoperative studies.

Echocardiographic studies. Echocardiographic studies were obtained before and after MVR in 21 patients with chronic, isolated MR who underwent echocardiography preoperatively and postoperatively. Demographic data are shown in Table 1. Some of the data from patients included in this series have been previously reported (2). No patient had evidence of obstructive coronary artery disease or a wall motion abnormality. Thirteen patients underwent MVR without preservation of the subvalvular structures and had transection of the chordae. Eight patients underwent MVR with preservation of chordal structures, using previously described surgical techniques (2). All postoperative echocardiographic studies were performed within 2 weeks of the operation. All patients were in normal sinus rhythm at the time of the preoperative and postoperative study.

Patient groups. Patients were classified into two groups based on the postoperative change in LV end-diastolic dimension. Group I patients experienced a reduction in LV end-diastolic dimension >5 mm or achieved a normal end-diastolic dimension postoperatively, or both. Group Ia patients (n = 8) underwent MVR with chordal preservation, and group Ib patients (n = 7) underwent MVR with chordal transection. Patients were assigned to group II (n = 6) if they did not exhibit a postoperative reduction in LV end-diastolic dimension >5 mm.

Mean systolic pressure. Mean systolic pressure was used as a surrogate for end-systolic pressure and was calculated using the method validated by Rozich et al. (2) in both normal patients and patients with MR. Mean arterial pressure (MAP) was calculated as

\[ \text{MAP} = (\text{Systolic blood pressure} + 2[\text{Diastolic blood pressure}])/3. \]

Echocardiographic measurements. Two-dimensional echocardiograms were recorded using standard techniques, on commercially available equipment (12). Two-dimensionally derived M-mode echocardiographic measurements of the minor axis dimension (LVID) and wall thickness (Th) were made. The LV chamber area was measured from freeze-framed two-dimensional recordings. These data were used to calculate LV volume using the Teichholz method (Equation 1). Measurements of dimension and wall thickness and calculations of volume were made at end-diastole and end-systole. Ejection fraction was determined using Equation 2. Left ventricular wall stress was calculated using Equation 4.

Statistics

Data are presented as the mean value ± SEM. Preoperative and postoperative changes in study variables were analyzed for statistical significance using the paired Student t test. Differences were considered significant at \( p < 0.05 \). Differences in baseline clinical characteristics between groups were analyzed by the unpaired Student \( t \) test.

Results

The preoperative and postoperative LV dimension, volume, EF and wall stress data are presented in Table 2. There were

<table>
<thead>
<tr>
<th>Table 1. Preoperative Clinical Data</th>
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<tr>
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<tr>
<td>Age (yr)</td>
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<tr>
<td>Group Ia  57 ± 6</td>
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<tr>
<td>Group Ib  60 ± 6</td>
</tr>
<tr>
<td>Group II  59 ± 5</td>
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</tr>
<tr>
<td>II                               3</td>
</tr>
<tr>
<td>III                              3</td>
</tr>
<tr>
<td>IV                               2</td>
</tr>
<tr>
<td>Pathogenesis of MR</td>
</tr>
<tr>
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</tr>
<tr>
<td>Mitral prolapse                  1</td>
</tr>
<tr>
<td>Degenerative                     3</td>
</tr>
<tr>
<td>Infective endocarditis           1</td>
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</tbody>
</table>

Data are presented as mean value ± SEM or number of patients. F = female; M = male; MR = mitral regurgitation, NYHA = New York Heart Association.
no significant differences in preoperative echocardiographic variables between the three groups of patients. There was a trend toward a larger heart size and lower EF in group II patients. In addition, preoperative values in group II patients of end-systolic dimension, end-systolic volume, fractional shortening and EF approached limits suggested by previous studies to predict suboptimal postoperative outcome (1,13–16). However, the differences between groups did not reach statistical significance, and identifying preoperative indices of postoperative outcome was beyond the aim of the current study.

By definition, patients in group I exhibited a significant \( p < 0.05 \) reduction in LV end-diastolic dimension and volume, whereas those in group II had no change in dimension or volume. In group I, the EF change depended on the nature of MVR. In group Ia, where patients underwent MVR with chordal preservation, there were no significant changes in EF after MVR, such that EF was \( 64 \pm 2\% \) preoperatively and \( 63 \pm 3\% \) postoperatively. In this group, end-diastolic dimension and volume fell significantly (by definition) and end-systolic stress fell significantly from \( 56 \pm 6\) g/cm\(^2\) preoperatively to \( 40 \pm 7\) g/cm\(^2\) postoperatively \( p < 0.05 \). In contrast, for the patients undergoing MVR with chordal transection (group Ib), EF fell from \( 64 \pm 2\% \) to \( 50 \pm 4\% \) \( p < 0.05 \), but despite the significant decrease in end-diastolic dimension, there was no significant change in end-systolic dimensions, volume or stress. In group II, the EF also fell significantly, but in contrast to group I, end-systolic dimension, end-systolic volume and end-systolic wall stress increased significantly \( p < 0.05 \).

The average values for end-diastolic dimension, EF and end-systolic wall stress in each patient group are plotted in Figure 2. Data from the three patient groups were superimposed on the length-force-shortening relations that were derived from the mathematic model. Group Ia patients showed a decline in end-diastolic dimension and end-systolic stress, and the line connecting the preoperative and postoperative values paralleled the normal EF curve with no change in EF. Group Ib patients exhibited a decline in end-diastolic dimension and a fall in the EF; end-systolic wall stress did not change significantly. Group II patients had no change in end-diastolic dimensions and a fall in the EF; end-systolic stress increased. These length-force-shortening coordinates from our three groups of patients closely approximate those predicted by the mathematic model and therefore support the hypothesis.

### Discussion

Mitrval regurgitation burdens the LV with a volume load that leads to a series of compensatory myocardial and circulatory adjustments (13). With acute volume overload, the ventricle uses its preload reserve and the total stroke volume increases through the Frank-Starling mechanism. In addition, the low resistance runoff into the left atrium contributes to an increased EF and a decrease in systolic volume; according to the law of Laplace, systolic load (i.e., afterload) declines. Thus,
in acute MR, an increased EF (and increased total stroke volume) occurs as a result of an increase in LV preload in association with a decrease in afterload.

The major change that occurs during the transition from acute to chronic MR is an enlargement of the ventricle (1,3,13,17). This comes about through slippage and rearrangement of myocardial fibers in association with the addition of new sarcomeres and the development of eccentric LV hypertrophy. As a result, preload at the sarcomere level returns toward normal and the systolic unloading that is characteristic of acute MR is gradually replaced by normal systolic wall stresses and a normal or high-normal EF (3,4). Thus, the enhanced total stroke volume, seen in chronic, compensated MR, is “mediated through a normal performance of each unit of an enlarged circumference” (3). As this new steady state develops, the small hyperkinetic chamber of acute MR is converted into a large compliant ventricle that is well suited to deliver a large stroke volume. Eventually, however, these compensatory mechanisms fail and surgical correction of the regurgitant lesion becomes necessary.

**Postoperative changes in ejection fraction and afterload.**

Mitral valve replacement is almost always followed by a reduction in the LV EF (1–6). This occurs, in part, as a consequence of the transection of the chordae and disruption of the mitral subvalvular apparatus (18–20). As a result, geometric and related functional changes produce a fall in the EF. In some patients, particularly those with decompenated hearts, LV afterload increases after valve replacement and postoperative afterload excess contributes to the decline in EF (1). In contrast, patients with compensated ventricular function exhibit a postoperative reduction in LV chamber size with little or no change in afterload (1). Thus, changes in afterload are strongly influenced by changes in end-diastolic volume or dimension and by changes in EF. For any end-diastolic volume, a lower EF is associated with a larger end-systolic volume and, by virtue of the Laplace relation, a higher wall stress or afterload.

To illustrate how a change in LV end-diastolic volume (or dimension) interacts with a change in the EF to affect systolic wall stress, we developed a simple spherical mathematical model relating end-diastolic dimension and systolic stress over a wide range of EFs (Fig. 1). The family of curves shown in this figure illustrates the dependency of afterload on chamber size and EF. Data from the three groups of patients, when plotted with reference to this model, illustrate this dependency and validate the model (Fig. 2). Patients with little postoperative change in end-diastolic dimensions or volume (group II) exhibit a significant increment in systolic wall stress. In contrast, patients who experienced a postoperative reduction in chamber size had no change in afterload if the valve was replaced and the chordal structures disrupted (group Ib) and a reduction in afterload if the valve was replaced using chordal sparing techniques (group Ia). These data suggest that an afterload excess (caused by closure of the regurgitant leak) cannot explain the reduction in EF after MVR. The most likely cause of the fall in EF is the disruption of the subvalvular mitral apparatus (2). Thus, in our experience and in most published reports, chordal sparing techniques are associated with preserved ventricular function, unless there is an increase in afterload (Table 2). It appears, therefore, that the simple model shown in Figure 1 illustrates the length-force-shortening relations of MR under a variety of circumstances.

**Postoperative changes in preload.**

The current data support the conclusion that postoperative changes in EF were not consistently related to changes in preload. However, assessment of changes in preload is very difficult, especially for noninvasive studies. Therefore, the current data analysis should consider the possibility that postoperative changes in preload might influence EF in some patients. For example, in groups 1a and 1b there was a decrease in end-diastolic volume. Despite this fact, the EF in group 1b fell, whereas that in group 1a was unchanged. Thus, the change in end-diastolic volume (and preload) alone cannot explain these results. However, in group 1a, it is possible that the effect of a fall in end-diastolic volume was countered by a fall in afterload, and the result was no net change in EF. Conceivably, in group 1b, a fall in end-diastolic volume with no change in afterload could have caused a decrease in EF. However, we believe that the most important differences between these two groups is the presence or absence of chordal preservation and the associated changes in afterload. In group II, there were no changes in end-diastolic volume. Thus, preload was most likely unchanged, whereas EF declined. The decline was likely a consequence of chordal transection and an increase in afterload. Thus, the data presented in the current study suggest that a change in preload is not the dominant mechanism causing postoperative changes in EF. Rather, the changes in afterload and, importantly, the presence or absence of chordal preservation are the primary determinants of the postoperative EF after MVR.

**Clinical implications.**

End-diastolic dimension and EF were used in an attempt to develop a practical model that can be applied easily in any clinical echocardiography laboratory. These variables are widely used and are generally included in echocardiographic reports. Thus, directional changes in systolic wall stress can be estimated from clinical measures of chamber size and EF. The model is limited, however, by the assumption of a constant LV mass and systolic pressure. For this reason, our model cannot be used to determine absolute levels of wall stress for different patients and cannot be applied to postoperative studies that are done more than a few weeks after the operation. In the present study the model was applied to patients with chronic MR to illustrate the changes in afterload that occur as a result of MVR. Under these circumstances, LV mass and systolic pressures are relatively constant and the major determinant of afterload is chamber size and EF.

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References